Introduction to Computing

Mathematics for Computing

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1. The Number Systems
2. Binary Arithmetic
3. Boolean Algebra

Decimal, Binary, Octal, Hexadecimal

Given the number $x_k x_{k-1} \cdots x_1 x_0$ represented in base b, its decimal value is:

$$\sum_{i=0}^{k} x_i * b^i.$$

Number conversions

	To decimal	**Multiplier**
Binary [0-1]	$b_k \cdots b_7\ b_6\ b_5\ b_4\ b_3\ b_2\ b_1\ b_0$	
	$\cdots 2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$	Power of 2
Octal [0-7]	$b_k \cdots \overline{b_7\ b_6}\ \overline{b_5\ b_4\ b_3}\ \overline{b_2\ b_1\ b_0}$	
	$\cdots o_1 * 8^1 \quad o_0 * 8^0$	Power of 8
Hexadecimal [0-F]	$b_k \cdots \overline{b_7\ b_6\ b_5\ b_4}\ \overline{b_3\ b_2\ b_1\ b_0}$	
	$\cdots h_1 * 16^1 \quad h_0 * 16^0$	Power of 16

Number conversions

Decimal 6.625 equals to 110.101 in binary.

Converting the integer part 6 to binary:

Integer	Operation	Quotient (Integer)	Remainder
6	6 / 2	3	0
3	3 / 2	1	1
1	1 / 2	0 [STOP]	1

↑

Converting the fractional part 0.625 to binary:

Fraction	Operation	Fraction	Integer
0.625	0.625 * 2	0.25	1
0.25	0.25 * 2	0.5	0
0.5	0.5 * 2	0 [STOP]	1

↓

Arithmetic operations in binary

Addition	**Subtraction**	**Multiplication**	**Division**
$0 + 0 = 0$	$0 - 0 = 0$	$0 \times 0 = 0$	$0 \ / \ 0 =$ NA
$0 + 1 = 1$	$0 - 1 = 1$ (borrow 1)	$0 \times 1 = 0$	$0 \ / \ 1 = 0$
$1 + 0 = 1$	$1 - 0 = 1$	$1 \times 0 = 0$	$1 \ / \ 0 =$ NA
$1 + 1 = 0$ (carry 1)	$1 - 1 = 0$	$1 \times 1 = 1$	$1 \ / \ 1 = 1$

Note: The carry bit and borrow bit are required whenever an overflow and underflow happen, respectively.

Representation of unsigned integers

Binary	Decimal
00000000 00000000	0
00000000 00000001	+1
00000000 00000010	+2
00000000 00000011	+3
...	...
01111111 11111111	+32767
10000000 00000000	+32768
10000000 00000001	+32769
...	...
11111111 11111111	+65535

Note: The number of bits is fixed (depends upon the architecture).

Representation of signed integers

Binary	Decimal
1 0000000 00000000	-32768
...	...
1 1111111 11111101	-3
1 1111111 11111110	-2
1 1111111 11111111	-1
0 0000000 00000000	0
0 0000000 00000001	+1
0 0000000 00000010	+2
0 0000000 00000011	+3
...	...
0 1111111 11111111	+32767

Note: (1's complement of x) $+ 1 =$ 2's complement of $x = -x$, wherein 1's complement flips the bits in x.

Bitwise shift

```
i = 11
i >> 1 = 5
i <<2 = 44
```

Bitwise shift

```
i = 11
i >> 1 = 5
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```
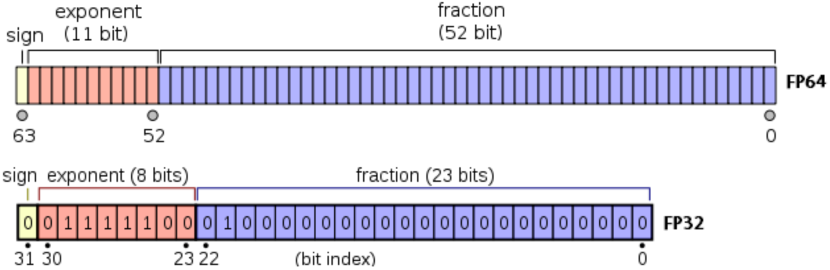

Decimal	Binary							
11	0	0	0	0	1	0	1	1
5	0	0	0	0	0	1	0	1
44	0	0	1	0	1	1	0	0

Floating point representation



Basics

Boolean algebra is a mathematical area that deals with operations on logical values with binary variables.

Boolean variables are represented as binary numbers to represent truthfulness, i.e., 1 denotes TRUE and 0 denotes FALSE.

De Morgan's theorems

- not (B_0 and B_1 and ... and B_n) = (not B_0) or (not B_1) or ... or (not B_n), where each B_i is a Boolean expression.
- not (B_0 or B_1 or ... or B_n) = (not B_0) and (not B_1) and ... and (not B_n), where each B_i is a Boolean expression.

Note: The original theorems are based on the union and intersection of sets.

Let's solve problems

Prove that $(n << 3) + (n << 1)$ will perform left rotation of the decimal digits in n by one position with zero padding for positive integers n. Note that $<<$ denotes bitwise left shift operation.

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Proof.

The expression $(n << 3) + (n << 1)$ reduces to the following:

$$(n * 2^3) + (n * 2^1) = 10 * n.$$

This is nothing but left rotation of the decimal digits in n by one position with zero padding. □

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Prove that if n is a power of 2 then n & $(n-1)$ is 0. Note that & denotes bitwise AND operation.

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Prove that if n is a power of 2 then n & $(n–1)$ is 0. Note that & denotes bitwise AND operation.

Proof.

If n is a power of 2, it can be written as follows:

$$1 * 2^k + 0 * 2^{k-1} + 0 * 2^{k-2} + \cdots + 0 * 2^1 + 0 * 2^0.$$

So, $n - 1$ can be written as follows:

$$1 * 2^{k-1} + 1 * 2^{k-2} + \cdots + 1 * 2^1 + 1 * 2^0.$$

Hence, n & $(n–1)$ will reduce to 0 as every pair of bits will differ. □

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Prove that for any arbitrary pair of bits A and B, the following will hold: not (A and B) = (not A) or (not B).

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Prove that for any arbitrary pair of bits A and B, the following will hold: not (A and B) = (not A) or (not B).

Proof.

This simply follows as a special case of one of the De Morgan's theorems. However, we can also prove this with truth tables as follows.

A	B	not (A and B)	(not A) or (not B)
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Hence, not (A and B) = (not A) or (not B). □

Homework

Prove or disprove the following statements:

- The operation $n = n \wedge (1 << p)$ will toggle (flip) the bit in the position p of the integer n. Note that $\wedge$ and $<<$ denote bitwise XOR and bitwise left shift operation, respectively.
- The binary number $[xy] \ldots 3k$ times is always divisible by 7, where x and y are any arbitrary bit (0 or 1).
- The Boolean expression (A and (A or B)) and (not A) gets always evaluated as False.